Communication for maths

Term 2 – week 8: On the presentation of series

Introduction

- These slides illustrate certain aspects relating to the presentation of curve sketching.
- Some relate to the presentation of all maths in general and some are specific to the presentation of curve sketching.
- Not all aspects of mathematics communication that we have studied so far will be presented here.
- Revise the slides on the previous topics where appropriate.

Use of brackets in summation expressions

- Let us have the series

$$1^2 + 1 + 2^2 + 1 + 3^2 + 1 + \cdots + n^2 + 1$$

- Which of the following represents this series?

a)	b)
$F(n) = \sum_{k=1}^{n} k^2 + 1$	$F(n) = \sum_{k=1}^{n} (k^2 + 1)$

Use of brackets in summation expressions

- Of the following expressions, which is/are confusing and which is/are unambiguous?

c)	d)
$F(n) = \sum_{k=1}^{n} (k^2 + 2k)$	$F(n) = \sum_{k=1}^{n} k^2 + 2k$
e)	**f)**
$F(n) = \sum k^2 + 2kr$	$F(n) = \sum 2kr + k^2$

Proof of summation formulae

Reminder

- The summation formulae for arithmetic and geometric progressions are respectively:

$$S_n = \frac{n}{2}[2a + (n-1)d] \quad \text{and} \quad S_n = \frac{a(1-r^n)}{1-r}$$

where all letters have their usual definitions.

Proof of summation formulae

Reminder

- When proving these formulae don’t forget the formality of the presentation of proofs:

Claim: - - - -

Proof

- - - -

- - - -

End of proof (or Q.E.D. or ■)

Selected methods in series.

Know your audience: Telescoping sums

- If you are writing a text book, i.e. a book for students to learn from, then crossing out terms as shown on the next slides is ok.

- The reason is that textbooks are aimed at a student audience. Textbooks are designed to help students learn mathematics they don't know, so a certain leeway is acceptable in the presentation of mathematics.

Selected methods in series.

$$\sum_{r=2}^{n}(r^2+1)r! \equiv \sum_{r=2}^{n}[f(r)-f(r-1)]$$

$$\equiv f(n)-f(n-1)$$

$$+f(n-1)-f(n-2)$$

$$+f(n-2)-f(n-3)$$

....

....

$$+f(3)-f(2)$$

$$+f(2)-f(1)=f(n)-f(1)$$

Showing cancellation of terms by crossing terms out.

This is ok for textbooks whose aim is to teach.

Selected methods in series.

Know your audience: Telescoping sums

- However, it is technically not correct to cross things out, or colour-highlight things, in mathematics (this is so when writing for a professional audience).

- Hence the correct approach would be to present the mathematics in the normal way, and then *explain the cancellation of terms* as shown below.

Hence $$\sum_{r=2}^{n}(r^2+1)r! \equiv \sum_{r=2}^{n} f(r)-f(r-1)$$

$$\equiv f(n)-f(n-1)$$

$$+f(n-1)-f(n-2)$$

$$+f(n-2)-f(n-3)$$

$$+\cdots-\cdots$$

$$+f(3)-f(2)$$

$$+f(2)-f(1).$$

Since all terms, apart from the first and last, cancel we obtain

$$\sum_{r=2}^{n}(r^2+1)r! \equiv f(n)-f(1).$$

Selected methods in series.

Telescoping sums: Exercise

- See separate pdf document (Moodle and/or MS Teams) for the exercise on telescoping sums.

- In this exercise the presentation of mathematics should be aimed at a professional audience not a student audience.

Selected methods in series.

AP and GP: Exercise

- Work on A.P.s and G.P.s involves only algebra, so there is nothing about the presentation of mathematics that is specifically related to APs and GPs.
- Therefore, all the standard rules of presentation apply.
- See separate pdf document (Moodle and/or MS Teams) for the exercises on AP and/or GP.

Maclaurin and Taylor series, and the word "expand"

- We know that we can say things like "Expand $(1 + x)^n$."
- We can now also say things like

$$\text{Expand } e^x \text{ as a Taylor series about } x = 1.$$

(see Friday lesson). In other words, any suitable function can be expanded, and the word "expand" is no longer just for binomials $(1 + x)^n$.

- At university, the word "expand" usually means Maclaurin or Taylor (or some other) series expansion.

Maclaurin and Taylor series, and the word "represents"

How to use the word "represents"

1. "The curve f(x) is positive" **No**

2. "The curve which represents f(x) is positive." **No**

3. "The curve which represents f(x) lies above the x-axis." **Yes**

Maclaurin and Taylor series, and the word "represents"

In terms of Maclaurin and Taylor series we have:

1. "The Maclaurin series is the function f(x)." **No**

This implies f(x) is identical to the Maclaurin series for all x.

2. "The function f(x) is represented by a Maclaurin series." **Yes**

This implies that a function can be *represented by* a Maclaurin series but need *not be equal to* its Maclaurin series.

Maclaurin and Taylor series, and the word "represents"

In terms of Maclaurin and Taylor series we have:

- For example

$$f(x) = e^{-1/x^2} \text{ for } x \neq 0$$

can be represented as

$$1 - \frac{1}{x^2} + \frac{1}{2!}\frac{1}{x^4} - \frac{1}{3!}\frac{1}{x^6} + \cdots$$

but

$$e^{-\frac{1}{x^2}} \neq 1 - \frac{1}{x^2} + \frac{1}{2!}\frac{1}{x^4} - \frac{1}{3!}\frac{1}{x^6} + \cdots$$